APPLIED MATHEMATICS IN HIGH SCHOOLS. SOME LESSONS FROM THE WAR.*

By William E. Breckenridge.

The war revealed to us America unprepared in mathematical training as in most other respects. A Colonel in Camp Upton reported as follows: "More men are required at Camp Taylor for the Field Artillery and no more can be found in this Camp who have the required mathematical training." What was this required training? The circular from Camp Taylor read: "Algebra through quadratics and Plane Geometry. The solution of triangles by Trigonometry is advised, but not required for entrance." As a matter of fact the essential algebra as revealed by the examinations set included hardly more than the use of formulas and ratio and proportion, while the goemetry was only mensuration. The original intention of the examiners for the Field Artillery School was to require all candidates to be graduates of high schools, but it was soon evident that this standard could not be maintained and an adequate supply of men secured. For the school at Fort Monroe, where men were trained for the Heavy or Coast Artillery, the requirement in mathematics was considerably more, including Plane Trigonometry.

When it became evident that there were not men enough in the country properly trained in mathematics for entrance to these artillery schools, the Government established a preparatory school near New York where a brief preliminary course could be taken before the examination for admission to the Heavy Artillery. At Camp Taylor, also, a preparatory course in mathematics was given consisting of very elementary practical mathematics before the men were considered properly fitted to undertake the real work of the Field Artillery.

In New York City Emergency Courses in Mathematics were given by the Y. M. C. A. and by Columbia University, designed to prepare men for the two schools of artillery in the shortest

* This paper was read at the Educational Congress in Albany, N. Y., in May.

possible time. It was found possible to do this work for most candidates in ten lessons of two hours each.

After the Armistice had been signed and plans were developed for reconstruction, one of the courses recommended for wounded soldiers was in The Use of the Slide Rule. A pamphlet was written under the direction of the Surgean-General's Office and the Board of Vocational Education giving ten lessons in very simple language such as could be understood by a soldier who had only had six grades of schooling. This work is now going on in the hospitals of the United States.

At Fort Monroe, under the able direction of Major Englehardt, an admirable system of instruction was developed. The daily program consisted of field work in the forenoon, lectures in the afternoon, and supervised study in the evening. Would not this arrangement delight the heart of any teacher of mathematics? The work of this school was very efficient. Yet when several French artillery officers inspected the school, one of them remarked, "You have a very fine kindergarten." Compared with the instruction in Frane, our work seemed very elementary.

Perhaps enough has been said to show that America was not prepared for war in the mathematical training of her young men. This is the first lesson in the teaching of mathematics to be learned from the war. It is one which those who are demanding less time for the teaching of mathematics would do well to ponder. In the new program for military training in our country it is evident that mathematics will have an important place.

A second lesson that the war has taught us is that much of our present subject matter in elementary mathematics is of no practical value and could profitably be replaced by material that is practical and at the same time has more value for cultural training. This material may be taken from arithmetic, algebra, geometry, and trigonometry. It includes real problems from everyday life for the purpose of motivation, illustration, and vocational training. It does not neglect thorough drill on fundamental operations necessary for the solution of these real problems. Teachers of mathematics know that the greater part of the time during the recitation must be spent in drill on the

mechanical operations until pupils acquire correct mathematical habits. This drill is not likely to be neglected. The emphasis at present should be in encouraging the use of practical problems. A real problem may be defined as a situation that happens to someone, somewhere, somtime. It is thus sharply distinguished from most of the problems in our algebras which never happen to anyone, anywhere, anytime. Of course, real problems should be selected that are within the experience of the particular pupils who are being instructed.

Following are some suggestions for improving our teaching of mathematics by the use of real applied problems:

1. The requirement in mathematics should be extensive enough to include a thorough working knowledge of practical mathematics. If mathematics is required through the ninth year, and is arranged so that the student may have an opportunity to take the essentials of algebra, geometry, and trigonometry, this can be accomplished. The new courses for Junior High Schools promise to help largely in this matter.

The early introduction of portions of physics into the curriculum, as is done at present in the first term of The Stuyvesant High School in New York City, opens a large field for practical problems.

2. The curriculum should be arranged so that the most useful mathematics may come first. It is evident that a controlling principle is necessary for the most effective course. The following is suggested: "What is the best mathematics for the pupil, no matter how long he remains in school?" Many pupils drop out during the first year of high school. What is the best mathematics that we can give them while they are with us? This seems to include the following topics, to be taught preferably by the Project Method: Real problems involving the use of the formula, how to make a formula, how to translate it into oral or written English, how to apply it to problems, how to solve it for any letter and interpret the results. Real problems involving the mensuration of all the common plane and solid geometrical figures.

Graphs of statistics, short methods, and checks will be included in this early work.

In an ordinary high school, ten weeks is sufficient for the

practical mathematics. If a longer course is desired, the slide rule and the practical part of trigonometry may be added. At the end of this course, the pupil will be equipped with sufficient mathematics to enable him to read mechanics' handbooks and elementary physics. If he must leave the day school he can attend science courses in an evening school. If he leaves at the end of ten weeks we have at least given him the mathematics that he needs most.

The second ten weeks may be used for equations of elementary algebra through quadratics. The test of whether a subject belongs in this part of the course is its use in solving an equation. The field of practical problems within the pupil's experience has been exhausted in the first ten weeks. In the second ten weeks, problems are concrete, but not many of them are real. This is the kind of applied problems found in the ordinary text-book in algebra. From this course will be omitted nearly all of factoring, fractions, radicals, and exponents.

Having finished a half year of work, the question is "Will it now be better for the student to finish the elementary algebra including the abstract work of factoring and fractions, or will it be better for him to have some of the splendid training in reasoning that comes in demonstrative plane geometry?" Most educators believe the latter plan is the better. It involves beginning plane geometry six months earlier than usual, but the subject matter may be made somewhat easier, if necessary, in the first course, by the omission of some theorems and the assumption of some others without proof. This plan has been tried in some of the largest schools in this country with successful results. The teaching of the subject six months earlier has not seemed to show any appreciable difference between the ability of these students and those on the former plan.

In the second year, first half, finish plane geometry.

In the second half, finish elementary algebra, including factoring, fractions, and radicals. Here again there will be some difficulty at first, because the pupils have not studied algebra for a year. At first it will seem like beginning algebra all over again. But the greater maturity of the pupils, and the fact that the poorer mathematical students have been eliminated by this time will soon show in the work of the class and render it very

easy to complete the elementary algebra within this half year. This arrangement of elementary mathematics does not retard the pupil who is going to college and it does give all students the mathematics that they need most in the order of its usefulness.

3. In all mathematics teaching practical problems should be used more largely for motivation and illustration. Where real problems are not available, concrete problems that are not real may be used. When a teacher begins to present a new subject, the approach should be by a problem involving that subject. For example, if it is desired to teach equations in x and y, present to the class at the beginning of the lesson a concrete problem whose solution involves an equation in x and y. As soon as the necessity for the new operation is evident, of course, most of the time of the recitation will be used in drill upon the mechanical operations involved until correct mathematical habits are developed. Most of our text-books could be improved in this respect. However, it is always possible for the teacher to supply the motivating problem, whatever the text-book.

For illustrative purposes we now have a large number of real problems in the fields of carpentry, housebuilding, metal-working, forestry, land measurements, physics, and mechanics. A new field has recently been opened involving Thrift, Savings, and Investment. Considerable literature has already appeared in this field. Evidently much of the Thrift of war time is to be continued in time of peace. Monographs on Thrift may be obtained from the Board of Vocational Education, Washington, D. C.

Teachers are usually ready and willing to use real problems if they are available. It is suggested that a card catalogue be developed classifying these real problems under their mathematical subjects and referring to the text where these problems may be found. In connection with a good library of reference books this catalogue will enable a teacher to find very quickly a real problem for motivating or illustrating the subject of the day's lesson.

4. Vocational courses in The Slide Rule, and the use of Surveying Instruments should be more largely organized. The Battery Commander's Ruler and The Musketry Ruler are instruments that can be utilized in the mathematics class room.

5. A method of teaching emphasized by the war was The Coöperative Plan under which the student works one week in school and the next in a shop, alternating throughout the course. Various modifications of this plan are in use for courses in Applied Mathematics. It works particularly well in surveying.

6. The Colleges and The College Entrance Examination Board should make courses in applied mathematics count definitely toward College Entrance.

7. In teaching Trigonometry, the practical part should be taught first with as much field work in surveying as can be arranged. The formulas may be assumed. Follow this with The Slide Rule, then complete the course by teaching the proofs of formulas, identities, and the solution of trigonometric equations.

8. In conclusion, while the greater part of our time as mathematics teachers will quite properly be spent upon the development of correct mathematical habits in our students, by means of drill on the mechanical operations, "problems without content," yet much can be done to improve the efficiency of our mathematics teaching by (1) the rearrangement of the curriculum with the controlling thought "What is the best mathematics for the student, no matter how long he remains in school?"—which means teaching the real applied problem first. (2) The increased use of practical problems for motivation, illustration and vocational training.

Let us not allow our zeal for practical problems to lead us to a neglect of the necessary parts of pure mathematics. There should emerge from any good mathematical course students who are well equipped with correct habits in mathematical calculation. But these habits do not need to be dissociated from application. Rather it is an advantage even on the side of training, to have mathematical laws associated with real problems.

In the struggle to make the world safe for democracy, what is more inspiring to a teacher of mathematics than to feel that he is not only keeping the standard high for the ten per cent. who go to college and become leaders, but he also has a part in making mathematics democratic, *i.e.*, of the most use to the most people?

TEACHERS COLLEGE,
NEW YORK CITY.